First European Conference on Earthquake Engineering and Seismology
(a joint event of the 13th ECEE & 30th General Assembly of the ESC)
Geneva, Switzerland, 3-8 September 2006
Paper Number:166

EVALUATION OF ENERGY DISSIPATED BY HYSTERESIS IN R/C SHEAR WALLS USING PARAMETERS OF DISPLACEMENT

Claudio OYARZO VERA[1]

SUMMARY

The energy dissipated by hysteresis in structures under reversible load patterns is one of the most widely used parameters to evaluate cumulative effect of load history (cumulative damage). It is used to identify different behaviors between elements whose failure is dominated by flexion or shear, but also many others phenomena.

However, it is necessary to evaluate this parameter to have a complete record of load-displacement history of each member of the studied structure.

This paper presents the results of tests made over reinforced concrete walls built with the same quantity of longitudinal reinforcement and different quantities of transversal steel. The objective of differences in reinforcement is to have an equal resistance to flexion, but different resistance to shear. These walls were tested under reversible loads until failure.

From analysis of loads-displacement records, it was possible to define a mathematical model which represents the behavior of energy dissipated by hysteresis using the cumulated ductility of displacements measured during testing.

There are two stages defined in this model. The first one represents -by a linear equation- the dissipation of energy by hysteresis when behavior of the wall is dominated by flexion.

The second stage -also linear- represents the dissipation of energy by hysteresis when the behavior of the wall is dominated by shear.

For this model it uses a parameter named γ, which corresponds to slope of second stage of the model, lower than slope of first stage. This parameter (γ) depends on the ratio ultimate shear v/s nominal shear, the load history and others.

With this work, it is possible to build a mathematical model which defines a relation between energy dissipated by hysteresis and cumulated ductility of displacement, which is easier to measure.

1. INTRODUCTION

The high seismic loads applied on structures in several situation force them to get in the non-elastic state, where the behavior of the structural elements depends on the energy dissipation capacity and high demands of ductility resistance. These irruptions in a level of deformation superior to the capacities of the structures are associated to a certain level of damage in its elements.

[1] Universidad Católica de la Santísima Concepción. Alonso de Ribera 2850. Casilla 297. Concepcion. CHILE
Email: coyarzov@ucsc.cl

One of the fundamental ideas of performance-based design postulates that by limiting certain response parameters it would be possible to control the damage produced on the structure. So, it would be necessary to establish limits for maximum or cumulative demands of several parameters and, also, to supply mechanical characteristics to the structure which help to control its response within the established limits [Teran-Gilmore, 1997].

There is a series of response parameters that can be associated to damage level. One of the more widely used parameters is the energy dissipated by hysteresis. It could represent very good the cumulative effect of the load history (cumulative damage).

Nevertheless, to be able to evaluate this parameter it is necessary to have a complete load-displacement record of each member part of the structure. That it is so, because, as it is well known, the dissipated energy could be evaluated by the area locked up inside the recorded hysteresis cycles.

This process is not always easy, because not all the cycles are closed. So, it is necessary to define some criteria to define the beginning and end of each cycle. In addition, these cycles have irregular geometries, which demand the implementation of numerical integration methods. Although this process is not complex, it can be very tedious and requires a postprocessing of data. Usually, the accuracy of this procedure is difficult to verify in a fast and simple way, because it is not easy to determinate at first sight, if the area calculated by integration method corresponds to the real area locked up by cycles.

This situation lead us to think: if it is possible to relate the hysteresis energy with some other easier to measure parameter, this relation would turned out to be a powerful tool to quantify the cumulative effect of load cycles.

2. PROBLEM DEFINITION AND OBJECTIVES

Following the last idea in the previous paragraph, this research tries to establish the bases of a mathematical model which allow the indirect measurement of hysteresis energy. This model suggests the use of displacement parameters, which are easy to measure and to predict in structures. The selected parameter corresponds to cumulated displacement ductility, it is the sum of all displacements registered after the elastic rank, normalized by the yield displacement of each structural element.

This parameter was chosen, because it represents in certain way the load-displacement history of each test, and it is capable to represent the deterioration of stiffness and resistance in structural element due to the cumulative effect of damage. In addition, it gives dimensionless values which allow comparison between elements with different geometric characteristics.

This work presents the results of a research made with a set of reinforced concrete walls, designed to allow a failure by shear previous to the failure by flexion, tested under cyclical load.

3. EXPERIMENTAL BASIS

The experimental basis which endorse this research corresponds to the data collected from 12 reinforced concrete walls tested in 1999 in the Universidad Técnica Federico Santa Maria in the context of a project funded by the “Fondo Nacional de Desarrollo Científico y Tecnológico” of the Chilean Goverment [Leiva, Bonelli et Al., 1999].

These walls were designed in order to obtain an initial resistance to shear high enough to develop yield in flexion, but simultaneously small enough so that the degradation allowed a failure by shear previous to the failure by flexion.

The walls were designed with a rectangular section, without border elements. The thickness of the walls was 10 (cm), the length was 80 (cm) and the height was 150 (cm). They were mounted on a rigid beam to provide embedding conditions and another rectangular (20x20 cm) beam was constructed at the top whose function was to transmit the lateral load to the wall (Figure 1).

The longitudinal reinforcement (flexion) was the same for all the walls. It was formed by three 12(mm) diameter bars triangularly arranged in each end of the section and a row of 8 (mm) diameter bars in the web of the wall. In addition, triangular stirrups were used to link the three 12 (mm) bars of the ends. In some cases, 6 (mm) diameter rectangular stirrups were used for linking the extreme bars with the first bar of the web in order to provide confinement to longitudinal reinforcement at ends.

Finally, the transversal reinforcement was different in each wall, to obtain different resistance to shear. For this, 5.5 (mm) diameter bars anchored to edge reinforcement with 180º hooks were used. Its spacing was variable to obtain different ratios of cross-sectional reinforcement.

Each wall was tested in a reaction frame anchored to the floor. The load pattern was applied to them on the top-beam by a double action load cell with 20 (T) of capacity.

Each sample was wired to measure displacements, angular deformations and loads. These data were automatically stored on a computer.

Walls M5 and M6A were loaded monotonically until the failure. All the other walls were tested under reversible load.

The load pattern used began with a series of three cycles of equal magnitude, then they progressingly increased until reaching the cracking by flexion. After this point, each load history series began with a first stage (degradation cycles) whose function is to degrade the resistance and stiffness of the element. This degradation cycles group was composed by four cycles which decrease one rate of 25%. The second stage (stabilization cycles) tried to stabilize the response of the walls, they included three cycles of equal magnitude to the first of the previous degradation cycle.

This series was repeated continuously, increasing the cycles amplitude and following the pattern already described until reaching the failure. Walls M1, M2B, M6B and M7B were tested with a similar load pattern shown in Figure 2. Wall M3B was tested with a load pattern where the stabilization cycles were omitted. Walls M2A, M3A, M4, M6C and M7A were tested with a load pattern with approximately the double or triple of cycles than the standard test in order to obtain a greater shear resistance degradation.

A summary of the most important characteristics of reinforcement appears in Figure 1 and Table 1. The Standard load pattern appears in Figure 2.

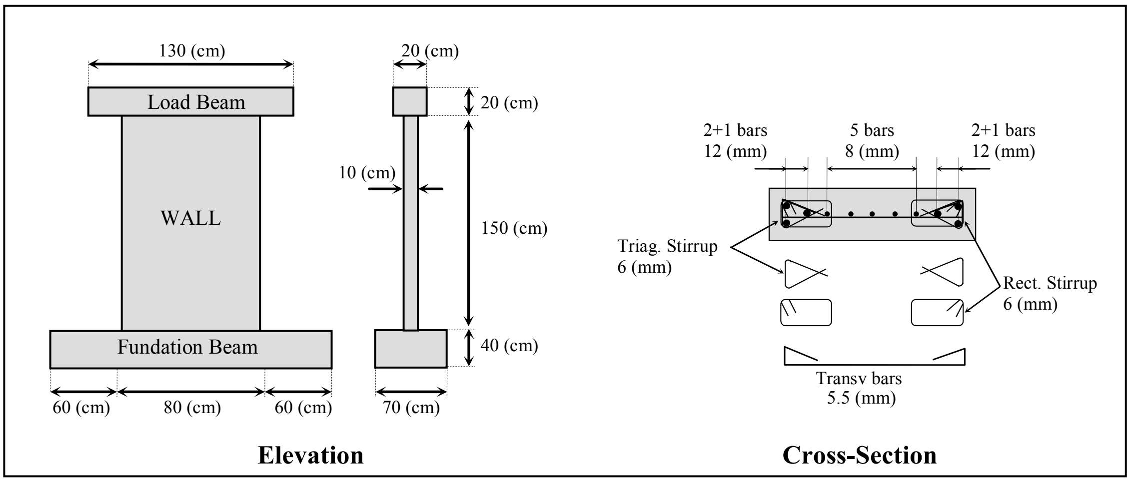

Figure 1: Walls' Geometry and Reinforcement Details.

Table 1: Walls' Reinforcement and Load Test Patterns

Wall	Edge Longitudinal Reinforcement	Web Longitudinal Reinforcement	Web Transversal Reinforcement	Transv. Reinf. Ratio %	Load Pattern
M1	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 8 (cm)	0.300	Standard
M2A	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 12 (cm)	0.200	Double Standard
M2B	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 12 (cm)	0.200	Standard
M3A	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 14 (cm)	0.171	Double Standard
M3B	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 14 (cm)	0.171	Only degradation Cycles
M4	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 25 (cm)	0.096	Standard with triple degradation
M5	3 bars of 12 (mm)	5 bars of 8 (mm)	No Reinforced	0.000	Monotonic
M6A	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 25 (cm)	0.096	Monotonic
M6B	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 25 (cm)	0.096	Standard
M6C	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 25 (cm)	0.096	Standard with triple degradation
M7A	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 20 (cm)	0.119	Double Standard
M7B	3 bars of 12 (mm)	5 bars of 8 (mm)	Bars of 5.5 (mm) @ 20 (cm)	0.119	Standard

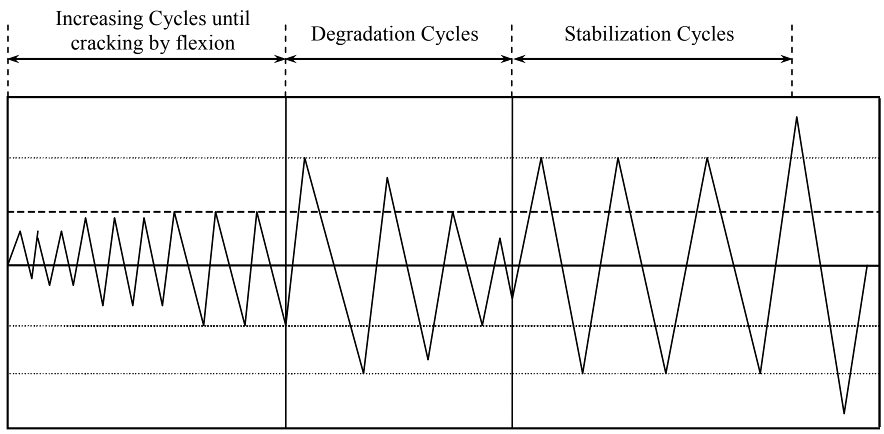

Figure 2: Standard Load Pattern

For every wall, a complete record of the response, in terms of the load-displacement cycles on the top of the wall was obtained. As an example, the records for wall M1 and M3A were in figure 3 and figure 4., both were tested under reversible load.

The types of failure of each wall are indicated in Table 2. In this table, the parameter u_y represents the displacement registered at the top of the wall at yield of the longitudinal reinforcement and μ_c corresponds to the cumulated displacement ductility of the wall, which is the sum of all maximal displacements registered at the top after the yield of the longitudinal reinforcement normalized by the yield displacement measured in each wall. Finally, ($u_y/u_{y\ mon}$) represents the ratio between the yield displacement of each wall normalized by the yield displacement of the reference wall tested under monotonic load (M6A).

Walls M5 and M6A, monotonically loaded, and Wall M6C, which showed an abnormaly type of failure (base sliding), will not be considered in the next analyses, because this phenomena are not included in this research.

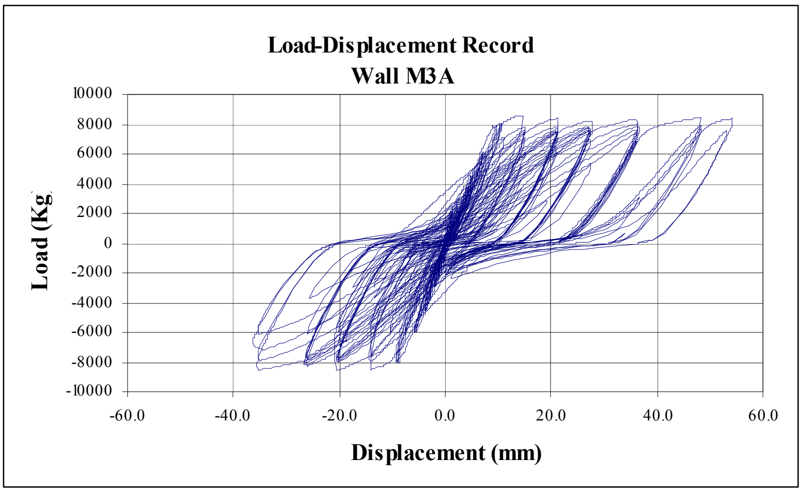

Figure 3: Load-Displacement Record Wall M3A

Table 2: Experimental Results and Observed Failure.

Wall	Transv. Reinf. Ratio (%)	u_y (mm)	$u_y/u_{y\ mon}$	μ_c	Observed Failure
M1	0.300	4.2	0.87	258.0	Flexion.
M2A	0.200	6.7	1.40	203.0	Shear - Diagonal Compression.
M2B	0.200	4.6	0.96	165.5	Shear - Diagonal Compression.
M3A	0.171	4.4	0.92	225.3	Shear - Diagonal Compression.
M3B	0.171	7.4	1.54	89.5	Shear - Diagonal Tension.
M4	0.096	4.7	0.98	351.8	Shear – Web Destruction
M5	0.000	10.3	2.15	4.8	Shear.(Monotonic Test)
M6A	0.096	4.8	1.00	19.1	Flexion (Monotonic Test)
M6B	0.096	6.8	1.42	125.8	Shear - Diagonal Compression.
M6C	0.096	--	--	--	Sliding.
M7A	0.119	4.4	0.92	285.8	Shear - Diagonal Compression.
M7B	0.119	6.2	1.29	148.5	Shear – Diagonal Compression and Tension.

4. EXPERIMENTAL RESULTS

From the records obtained for each wall, the cumulated displacement ductility for every instant until the wall failure point was calculated. This was made using the maximal displacement registered at the top of each wall at every single half-cycle normalized by the total displacement at the top of each wall when longitudinal reinforcement yields. This dimensionless value corresponds to displacement ductility reached in the half-cycle. The sum of all displacement ductility reached until that moment corresponds to cumulated displacement ductility. This is:

$$\mu_c = \sum_i \frac{u_{max\,i}}{u_y} \qquad (1)$$

where:

μ_c : Cumulated displacement ductility.

$u_{max\,i}$: Maximal displacement on the top at the half-cycle i.

u_y : Maximal displacement on the top at yielding condition.

Also, energy dissipated by hysteresis was quantified along all test. It was made calculating with numerical integration methods the area locked up by the cycles. This value was normalized in order of be able to compare results obtained for each wall.

The normalized parameter of energy chosen corresponds to the parameter used in the Damage Index of Park & Ang [Park & Ang, 1985], but it has been corrected using the term ($u_y/u_{y\ mon}$), which relates the cyclical test behavior with the monotonic test.

So, the normalized hysteresis energy is defined by the expression:

$$E_{H\,N} = \frac{\left(E_H \big/ F_y \cdot u_{mon} \right)}{\left(u_y \big/ u_{y\,mon} \right)} \qquad (2)$$

where:

$E_{H\,N}$: Normalized hysteresis energy.

E_H : Dissipated hysteresis energy.

F_y : Yield strength.

u_{mon} : Ultimate deformation reached due monotonic load test.

u_y : Yield displacement.

$u_{y\,mon}$: Yield displacement recorded on monotonic load test.

From these data it was possible to obtain graphs that relate energy dissipated by hysteresis with cumulated displacement ductility along all the test for each wall, as it appears in figure 4

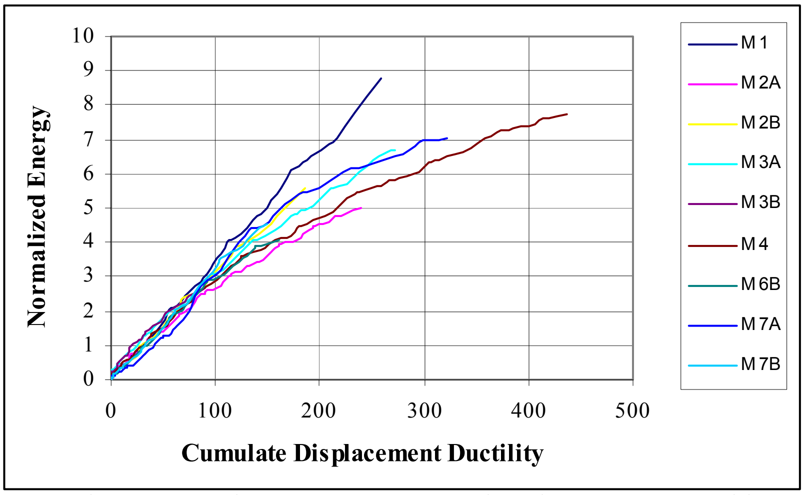

Figure 4: Normalized hysteresis energy and cumulative displacement ductility during tests.

5. RESULTS ANALYSIS

Figure 4 clearly shows how the curve energy-ductility of walls that failed by shear present as a common characteristic the diminution of slope when cumulated ductility increases. This phenomenon is associated to loss of energy dissipation capacity due to degradation by shear.

Considering that, the use of a bilinear function to model performance of walls that fail by shear is proposed. This kind of equation could very well reproduce the effect of degradation in stiffness and resistance. This model considers a first stage defined by a lineal equation, common for all walls. This equation is valid until a certain level of cumulated displacement ductility, where the shear damage became more important than flexural damage. This level of ductility has been named “Shear Cumulated Displacement Ductility” ($\mu_{c\ sh}$). After this point began a second stage, also linear, but with a different slope in each wall.

So, the proposed model is defined by equations:

First Stage:

$$E_{HN} = 0.0315 \cdot \mu_c \qquad 0 < \mu_c < \mu_{c\,sh} \qquad (3)$$

Second Stage:

$$E_{HN} = E_{H\,Fl} + \gamma \cdot (\mu_c - \mu_{c\,sh}) \qquad \mu_{c\,sh} < \mu_c \qquad (4)$$

where:

E_{HN} : Normalized hysteresis energy, defined in (2).
μ_c : Cumulated displacement ductility.
$\mu_{c\,sh}$: Shear cumulated displacement ductility (in this case $\mu_{c\,sh} = 80$).
$E_{HN\,Fl}$: Normalized hysteresis energy dissipated during first stage (in this case $E_{HN\,Fl} = 0.0315 \cdot \mu_{c\,sh}$).
γ : Second stage slope.

It must be notice that equations (3) and (4) only depend on displacement variables (μ_c , $\mu_{c\,sh}$) and on parameter γ which represents the slope of second stage in the model.

Using this model it has been obtained the following equations for each tested wall:

Table 3: Normalized Hysteresis Energy.

Wall	E_{HN} First stage	E_{HN} Second stage
M2A	$0.0315 \cdot \mu_c$	$0.0170 \cdot \mu_c + 1.1617$
M2B	$0.0315 \cdot \mu_c$	$0.0285 \cdot \mu_c + 0.2388$
M3A	$0.0315 \cdot \mu_c$	$0.0201 \cdot \mu_c + 0.9092$
M4	$0.0315 \cdot \mu_c$	$0.0169 \cdot \mu_c + 1.1694$
M6B	$0.0315 \cdot \mu_c$	$0.0211 \cdot \mu_c + 0.8351$
M7A	$0.0315 \cdot \mu_c$	$0.0206 \cdot \mu_c + 0.8689$
M7B	$0.0315 \cdot \mu_c$	$0.0306 \cdot \mu_c + 0.0708$

In figure 5 it is possible to see how second stage slopes (γ) are ordered naturally in three defined groups.

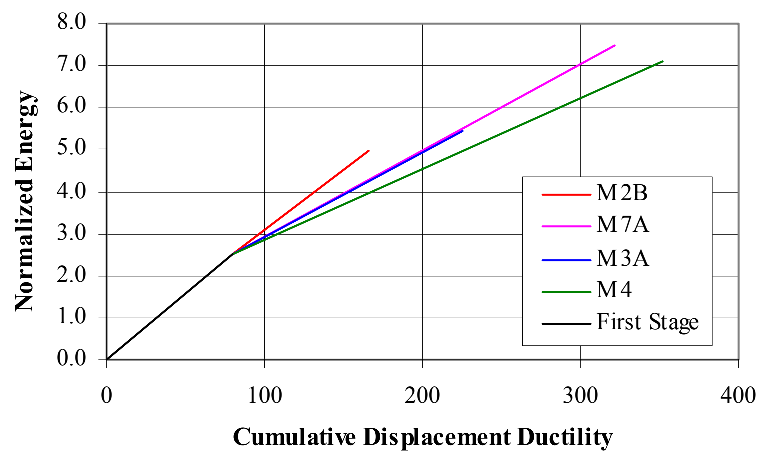

Figure 5: Bilinear Model. Representative Walls.

From the complete set of tested walls, only four of them have been chosen as representative (M2B, M2A, M4 and M7A), because they show a yield displacement very near the yield displacement recorded on monotonic reference test (M6A), that is, $u_y/u_{y\,mon} \approx 1$. This characteristic indicates that the design hypotheses (equal flexural resistance) has a satisfactory performance.

5.1 Determination of γ

In order to establish some law which allows to determine the value of γ, its relation with a series of structural parameters was analyzed. These parameters were:

- The ultimate shear resistance (experimental) normalized by the nominal shear resistance (theoretical) calculated in base of the Concrete Design Code ACI 318/99 [ACI 318, 1999], that is, V_u/V_n. This dimensionless parameter was selected, because it indirectly considers the transverse reinforcement ratio, which is the main variable that makes the difference between walls. In addition, it reflects the differences between the theoretical and experimental behavior.

- The ratio between number of stabilization cycles and the number of degradation cycles (*N.E.C./N.D.C.*) of the load pattern. This parameter was used to distinguish the different tests, load pattern and degradation level expected in the walls.

The values of these parameters are shown on table 4.

Table 4: Parameters associated to γ.

	Wall	Load Pattern	γ	$u_y/u_{y\,mon}$	Transv. Reinf. Ratio (%)	V_u/V_n	$\frac{N.E.C.}{N.D.C.}$
Group 1	M4	Triple Standard	0.0169	0.96	0.096	0.60	0.29
Group 2	M3A	Double Standard	0.0201	1.00	0.171	0.58	0.63
Group 2	M7A	Double Standard	0.0206	0.86	0.119	0.61	0.65
Group 3	M2B	Standard	0.0285	0.96	0.200	0.62	0.72

By a simple linear approach (Fig. 6 y Fig. 7), it is possible to obtain equations which represent the behavior of these parameters related with γ. These linear functions are showed on equations (5) and (6).

$$\gamma_1 = 0.587 \cdot \left(\frac{V_u}{V_n}\right) - 0.332 \qquad > \qquad 0 \tag{5}$$

$$\gamma_2 = 0.0327 \cdot \left(\frac{N.E.C.}{N.D.C.}\right) + 0.0028 \qquad > \qquad 0 \tag{6}$$

where:

V_u : Ultimate shear resistance recorded on the wall.
V_n : Nominal shear resistance calculated according to ACI-318/99.
N.E.C. : Number of stabilization cycles on the load pattern.
N.D.C : Number of degradation cycles on the load pattern.

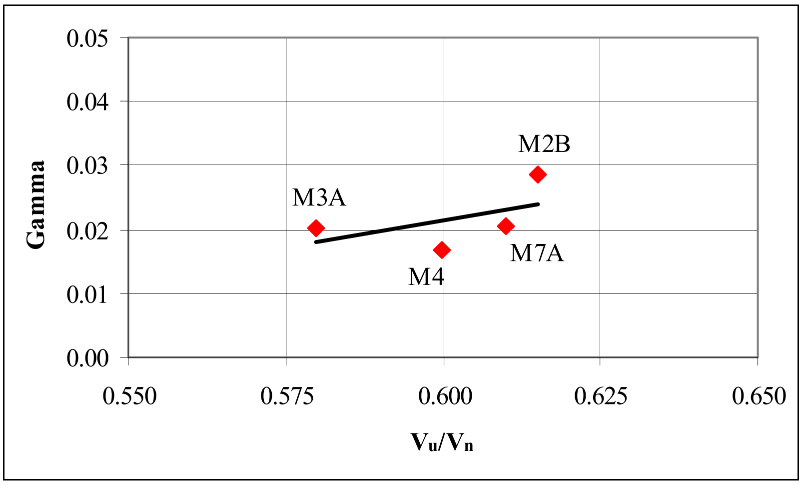

Figure 6: Relation V_u/V_n - γ

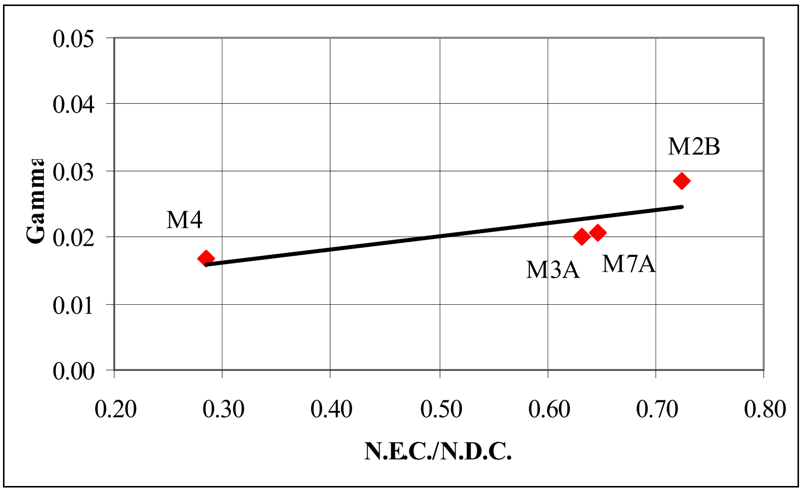

Figure 7: Relation *N.E.C/N.D.C* - γ

Finally, it is necessary to define an equation to estimate the value of V_u in order to obtain a complete energy prediction model. To achieve this, the use of the function submitted by professor Gilberto Leiva in 1999 is suggested [Leiva, Bonelli et Al., 1999]. This equation depends on structural variables and ductility, and it showed a very good performance with the wall tested in this research.

$$Vu = \begin{cases} 0.7\cdot\left[\left(1-\dfrac{\mu}{116}\right)\cdot\sqrt{fc'} + \rho_t\cdot f_y\right]\cdot b\cdot d & 0 < \mu < 50 \\ \left[0.4\cdot\sqrt{fc'} + 0.7\cdot\rho_t\cdot f_y\right]b\cdot d & 50 < \mu \end{cases} \tag{7}$$

where:

- μ : Average displacement ductility reached in both directions.
- fc' : Cylindrical resistance stress of concrete.
- f_y : Yield stress of reinforcement.
- ρ_t : Transverse reinforcement ratio.
- b : Wall thickness.
- d : Wall effective length.

However the data volume is little, it is possible to forecast a model to γ, using the equations (5) y (6) and a third equation which correspond to a linear combination of both equations.

$$\gamma_3 = 0.3\cdot\gamma_1 + 0.7\cdot\gamma_2 \qquad > \qquad 0 \tag{8}$$

On table 5 and figure (8), the results of the models to E_{HN} and, also, the experimental values measured of E_{HN} directly from the records are presented.

Table 5: Results of the model suggested to E_{HN}.

Wall	μ	f_c'	b	d	f_y	$\mu_{c\,sh}$	V_u	V_n	V_u/V_n	$\frac{N.E.C.}{N.D.C.}$	γ_1	γ_2	γ_3	$E_{HN}1$	$E_{HN}2$	$E_{HN}3$	E_{HN} Exp.
M2A	7.8	367	10	66	3200	95.2	8.01	13.4	0.59	0.63	0.019	0.023	0.022	5.06	5.52	5.38	4.55
M2B	12.7	370	10	66	3200	78	8.03	13.5	0.59	0.72	0.017	0.026	0.024	3.98	4.77	4.53	4.93
M3A	12.6	398	10	66	3200	91.7	7.79	13.2	0.59	0.63	0.015	0.023	0.021	4.84	6.02	5.67	5.40
M3B	11.8	389	10	66	3200	63.1	8.12	13.1	0.62	0.24	0.032	0.011	0.017	2.83	2.27	2.44	2.70
M4	14.6	370	10	66	3200	76.1	6.50	11.3	0.58	0.29	0.006	0.012	0.010	3.92	5.74	5.20	6.92
M6B	10.8	335	10	69.5	3320	21.9	6.64	11.5	0.58	0.70	0.007	0.026	0.020	1.40	3.36	2.77	4.06
M7A	15.5	318	10	69.8	3320	74.3	6.91	11.8	0.59	0.65	0.012	0.024	0.020	4.82	7.41	6.63	7.00
M7B	13.2	302	10	69.9	3320	8.6	6.79	11.6	0.59	0.71	0.012	0.026	0.022	1.91	3.93	3.32	4.55

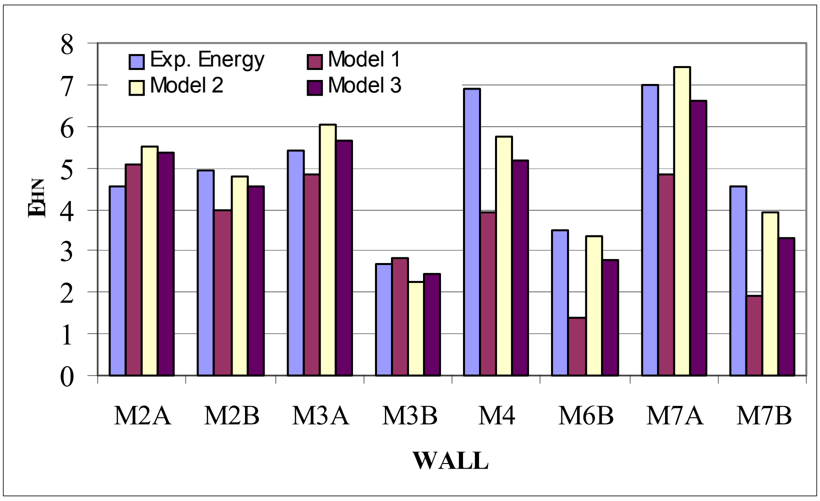

Figure 8: Results of energy models.

6. CONCLUSIONS

The analysis of experimental data allowes to conclude that it is possible to estimate the cumulative effect of cyclical actions by parameters displacement (ductility) instead of parameter of energy (hysteresis energy).

The study of correlation between these two parameters make possible to affirm, that in walls whose failure was dominated by shear is feasible to built a bilinear analytical model. This model should represent hysteresis energy dissipation based on structural variables (V_u, V_n, degradation level) and displacement parameters (μ_c , $\mu_{c\ sh}$). These variables are easier to measure and it is possible to perceive them at first sight. In addition, it has been established a relation between the second stage slope of the model (γ) and parameters as V_u/V_n and degradation level.

Nevertheless, the reduced data volume make necessary to continue these analyses, in order to refine the model. Specially it would be important to detect others variables that affect behavior of γ and to define equations which govern this relations.

7. ACKNOWLEGMENT

This paper has been written based on the thesis whereupon the author obtained his degree of Civil Engineer in the Universidad Técnica Federico Santa María [Oyarzo, 2003] and summarizes only part of the researched topics. The support of this institution in the development of this research is thanked. Specially, the author wants to thank the help and guidance of Professor Gilberto Leiva H.

Finally, it is important to acknowledge the assistance of the Universidad Católica de la Santísima Concepción in the elaboration of this paper.

8. REFERENCES

Instituto del Cemento y el Hormigón – Chile (1999), Código de Diseño de Hormigón Armado. Basado en ACI 318-99, *ICH, Chile.*

Leiva, G.; Bonelli, P.; Méndez, J.; Sanz, C.; Delgado, M. (1999), Resistencia al Corte de Muros de Hormigón Armado sometido a Altas Demandas de Ductilidad. *Informe Final Proyecto Fondecyt Nº 195039, UTFSM*, *Valparaíso - Chile.*

Oyarzo, C. (2003), Evaluación Del Daño En Muros De Hormigón Armado Sometidos A Altas Demandas De Ductilidad. *Memoria para optar al título de Ingeniero Civil, UTFSM*, *Valparaíso - Chile*.

Park, Y-J.; Ang, A. (1985) Mechanistic Seismic Damage Model For Reinforced Concrete, *Journal of Structural Engineering, 111, n° 4, 722-739.*

Teran-Gilmore, A. (1997), Energy Concept and Damage Indices, *Proceeding of Symposium in Honor of Vitelmo Bertero, Berkeley, California Jan. 31-Feb. 1, 1997. http://nisse.berkeley.edu/lessons/teran-gilmore.html*